Mathematical Modeling of Inrush Current in Power Transformers

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Abstract: Transformers are key component for electrical energy transfer in power system. Stability and security of transformer protection are important to system operation. Transformer inrush currents are high-magnitude, harmonic-rich currents generated when transformer cores are driven into saturation during energization. These currents have undesirable effects, including potential damage or loss-of-life to the transformer, protective relay disoperation, and reduced power quality on the system. Inrush current waveform is similar to a half-wave rectifier waveform, with very big amplitude up to 3000 ampere and stop after 3 second. This has always been a concern in the transformer and load power quality. In this paper, we will analysis and modeling of inrush current in transformer.

Keywords: Residual flux, Magnetizing current, Inrush current, MATLAB

Introduction:-
Transformer is a essential device for power transmission system. When a unloaded or lightly loaded transformer connect to a random supply, than it create large flux asymmetries and one or more core of the transformer can be saturated. This saturation results in high magnitude currents that are rich in harmonics and have a high direct current component. This current called inrush current or surge current. The ratio of inrush current to full load current can be 5 to 10 times greater. This current can create many problems in transformer like false operation of relays, potential damage in transformer etc.

We know that the rate of change of instantaneous flux in a transformer core is proportional to the instantaneous voltage drop across the primary winding. Or as started before, the voltage waveform is the derivative of flux waveform, and the flux waveform is the integral of voltage waveform. In a continuously operating transformer, these two waveform are phase shifted by 90°. Since flux is proportional to the magneto motive force(MMF) in the core, and the MMF is proportional to winding current, the current waveform will be in phase with the flux waveform and both will be lagging the voltage waveform by 90°.

Residual flux:-
When a transformer is de-energized, a permanent magnetization of the core remains due to hysteresis of the magnetic material. This is called residual flux. This residual flux is influenced by the transformer core material characteristics, core gap factor, winding capacitance, circuit breaker current chopping characteristics and other capacitance connected to the transformer.

When a transformer is energized the instantaneous magnitude of core flux at the instant of energization is the residual flux. The amount of offset to the sinusoidal flux generated by the applied voltage is dependent upon the point of the voltage wave where the transformer is energized. By Fig. 1

\[ \phi_{\text{peak}} = 2\phi_{\text{normal}} + \phi_{\text{residual}} \]

For the most severe case, where energization was at voltage zero as fig. 1, the peak transient core flux is more then two times higher than the normal core flux. The core has been driven far into saturation. This asymmetrical saturation result in the typical inrush current transient characterized by a high harmonic content and a direct current component.

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Analytical expression for inrush current:-

After a transformer is connected to current pass a transient mode and then reaches the study state mode. Voltage waveform, instant of closing the switch, amplitude and direction of the magnetic residual are the factors that can last the transient mode. The primary current with secondary open circuit may rise to several times the rated current of the transformer. Ignoring the winding’s resistance value, the relationship between the voltage and flux is describe following

\[ E_m \sin \omega t = N_1 \frac{d\phi(t)}{dt} \]  
\[ = 2 \]  

Where \( N_1 \) = The number of turns in primary winding.

\[ \phi(t) = \text{Flux in primary} \]

So flux can be obtained as

\[ \phi(t) = -\frac{E_m}{\omega N_1} \cos \omega t + c = -\varphi_m \cos \omega t \]  
\[ = 3 \]

Where \( c = \) constant and

\[ \frac{E_m}{\omega N_1} = \varphi_m \]  
\[ = 4 \]

When \( t = 0 \) and \( \phi(t) = \phi(0) = \varphi \), then by equation 3

\[ \phi(t) = -\varphi_m \cos \omega t + \varphi_r + \varphi_m \]  
\[ = 5 \]

Where \( \varphi_r = \) the remanence in the iron core.

Now the inrash current represents as

\[ i_m = \frac{1}{L_{in}} \int E_m \sin \omega t dt \]  
\[ = 6 \]

During the period of transient inrush current, since the transformer’s core normally enters a state of saturation, the magnitude of inductance is reduce, the current increases quickly due to the decrease in inductance. This phenomenon has some dangerous effects. Instant in time when voltage is zero during continuous operation. This is the point in time where both flux and winding current are at their negative peaks, experiencing zero rate of change \( (d\phi/dt = 0, di/dt = 0) \). As the voltage build to its positive peak, the flux and current waveforms build to their maximum positive rate of change, and on upward to their positive peaks as the voltage descends to a level of zero, as shown in fig. 2

Fig. 1: Core flux showing energization case of residual condition

Fig. 2: Flux and winding current at their negative peaks

Where \( e = \) Voltage

\[ i = \text{Coil current} \]

\[ \varphi = \text{Magnetic flux} \]

Modeling of inrush current:-

We can use, MATLAB for modeling of inrush current in transformer. The transient was simulated using a set of elements from the MATLAB library including a three-phase two-winding saturable transformer of the same specifications, three single-phase sinusoidal voltage sources each of 11000 V, measuring and displaying devices to monitor the in-rush current and voltage waveforms as shown in figure 3.
Figure 4 to 10 shown the simulink result and these compare favorably with those obtained using proposed model.

Figure 4, 5 and shown the inrush current in power transformer. In figure 4 shown inrush current in phase A which has drawn 1900 Amps current. Figure 5 shown inrush current in phase B which is 1425 Amps and figure 6 shown inrush current in phase C which is 410 Amps these all currents are 8 time higher then rated value of transformer.

Figure 4 Inrush current in phase A

Figure 5 Inrush current in phase B

Figure 6 Inrush current in phase C

Figure 7 shows the flux in all three phase. Figure 8, 9 and 10 shown the harmonics generated in transformer.

Phase A contain 25.7 % THD (Total Harmonic Distoation) as shown in figure 8. Phase B has 39.12 % THD as figure 9 and figure 10 shown that phase C has 81.38 % of THD. These harmonics are very harmful for system.

Figure 8 Harmonics in phase A

Figure 8 Harmonics in phase B

Figure 8 Harmonics in phase C

Conclusion:-

This paper presents a mathematical analysis for an unloaded saturated transformer and simulates a transformer to found inrush current in transformer when the transformer is connected to the power supply. We can see that residual current has a main place in this procedure. By this model, we can see, that inrush current drawn by a transformer is reach 8 times to its full load current. So this inrush current is very harmful for transformer.

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